Subjective Mortality, Investment and Annuitization over the Financial Life-Cycle

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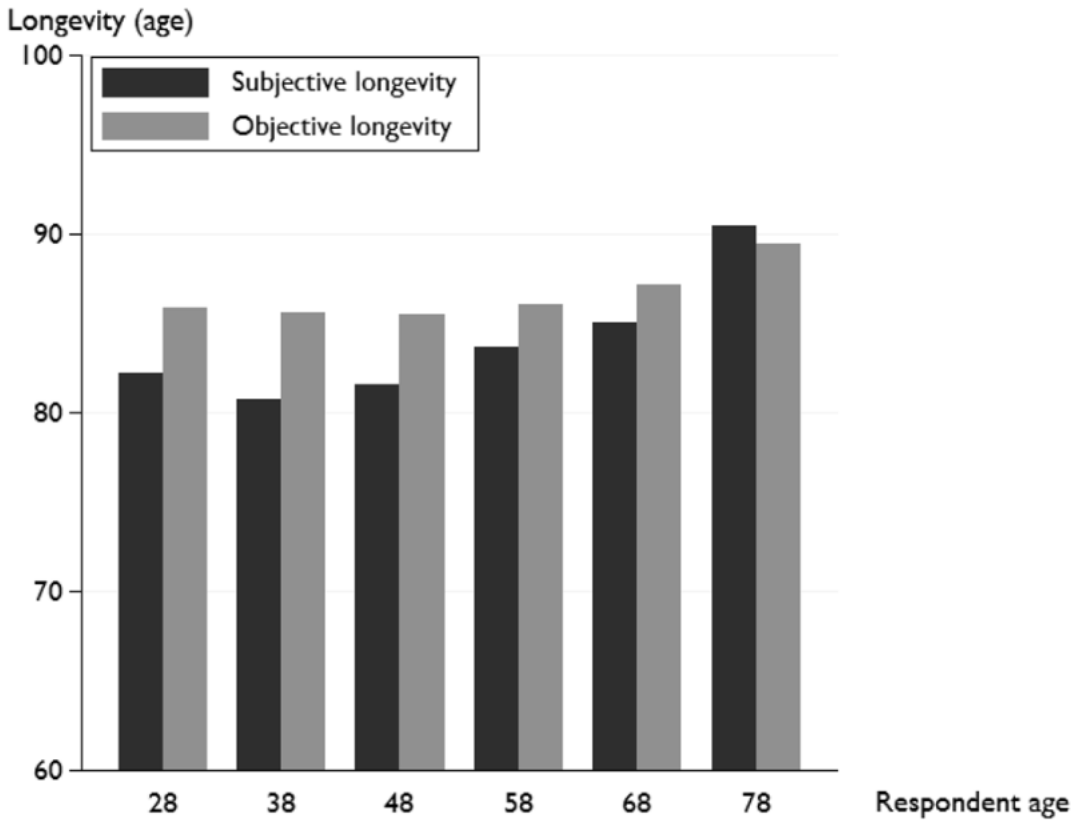
Problem Statement & Key Findings

Observation

Individuals have *subjective* survival beliefs which differ from *objective* survival probabilities derived from actuarial and demographic data



Source: Heimer et al. (2019)
Female survey respondents
Objective longevity: life table of the US Social Security Administration (SSA)

Problem Statement & Key Findings

Problem 1: Subjective mortality

How do we measure subjective mortality?

Answer:

We use stated subjective life expectancy e_x^i from U.S. Survey of Consumer Finances (SCF), and scale objective mortality via

- hazard-rate μ_x
- survival probability p_x

Problem 2: Life-cycle model

Does subjective mortality explain:

- under-saving prior to retirement
- slow wealth decumulation post-retirement
- under-annuitization?

Answer:

These stylized facts are only very marginally explained by subjective mortality

Related Literature

Evidence

Several studies, in different countries and over time, repeatedly demonstrate discrepancies between subjective survival beliefs and objective survival probabilities

Article	Panel Data	Years	Life Table
Gan et al (2005)	AHEAD	1993	
Puri & Robinson (2007)	SCF	1995,1998,2001	NIH
Salm (2010)	HRS	2000,2002	CDC
Elder (2013)	HRS, AHEAD	1992,1994,1996 1998,2000,2002,2004	CDC
Post & Hanewald (2013)	SHARE	2006,2007	HMD
Heimer et al (2019)	Own Survey		SSA
O'Dea & Sturrock (2021)	ELSA	2013	ONS

Other Results: Mortality

- Bounded rationality, limited information or cognition when estimating probabilities (Simon, 1955)
- Different people have different abilities to estimate lifespan (Hamermesh, 1985)
- People can estimate survival reasonably well based on health behaviours, e.g. smoking (Gan et al, 2005; Smith et al, 2001)
- Subjective survival beliefs can serve as predictors of actual mortality (Hurd & McGarry, 2002)
- Subjective survival beliefs can serve as predictors of longevity risk (Post & Hanewald, 2007; Perozek, 2008)

Other Results: Financial Decisions

Subjective survival beliefs can partially explain:

- household's stock market participation (Puri & Robinson, 2007)
- voluntary retirement age (Hurd et al, 2004; Van Solinge & Henkens, 2009)
- bequests (Gan et al, 2015)
- under-saving pre-retirement; slow wealth decumulation post-retirement (Heimer et al, 2019; Wu et al, 2015)

Other Results: Annuity Puzzle

Explanations for the "annuity puzzle"
(lower-than-optimal demand for annuities):

- Bequest motive
- Cost of annuities relative to actuarially fair price
- Social security and defined benefit pensions
- Hyperbolic discounting
- ⋮
- Value of option to delay irreversible annuitization (and participate in stock market)
- Framing effects
- Financial literacy
- Habit formation
- ⋮

- Subjective survival beliefs
  (O'Dea & Sturrock, 2021; Wu et al, 2015; Bateman et al, 2018)

Our Key Contributions

Subjective mortality

We estimate subjective survival beliefs

- at a full spectrum of adult ages (20 and above)
- benchmarked to objective life table probabilities
- scaled using survey respondents' reported subjective life expectancy

Life-cycle model

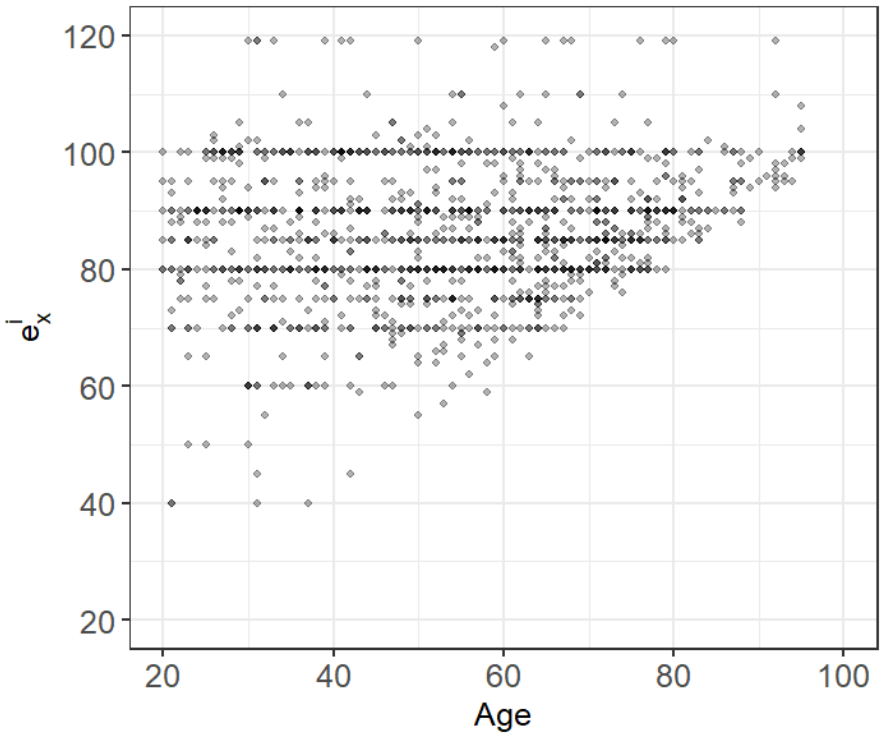
We implement a full life-cycle model with

- consumption and portfolio decisions
- stochastic stock returns
- stochastic wages, correlated to stock returns
- annuitization decision at retirement

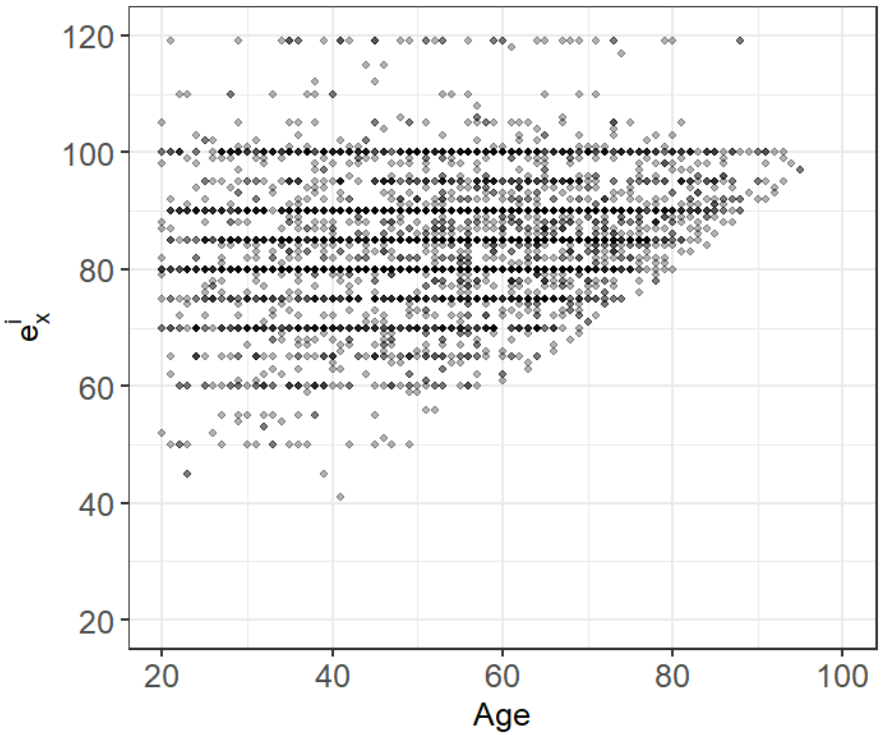
Data: U.S. Survey of Consumer Finances (2019)

Scatterplots of subjective expected age at death vs. age

Female respondents



Male respondents



Data: Characteristics of Survey Respondents (SCF, 2019)

Statistic	N	Mean	St. Dev.	Min	Max
Age	5,777	53.22	16.24	18	95
Subjective expected age at death					
without truncation	5,777	85.33	11.27	40	150
with truncation	5,777	85.21	10.75	40	119

- Subjective expected age at death is the age at which respondents believe that they will die
- If this age is greater than 119, it is truncated at 119
- $\omega = 119$ is the maximum age in a corresponding life table (U.S. Social Security Administration, 2019)

Model: Notation

	Objective	Subjective
1-year survival prob. at age x	$0 < p_x < 1$	$0 \leq p_x^i \leq 1$ for individual i
k-year survival prob. at age x	${}_k p_x = \prod_{j=0}^{k-1} p_{x+j}$	${}_k p_x^i = \prod_{j=0}^{k-1} p_{x+j}^i$ for individual i
Life expectancy	$e_x = \sum_{k=1}^{\omega-x} {}_k p_x$	$e_x^i = \sum_{k=1}^{\omega-x} {}_k p_x^i$

Subjective Mortality: Hazard (μ_x)-scaling

Hazard rate μ_x

$$p_x = \exp\left(-\int_0^1 \mu_{x+\tau} d\tau\right)$$

Hazard scaling

$$\mu_x^i = \gamma_i \mu_x$$

Subjective life expectancy

$$e_x^i = \sum_{k=1}^{\omega-x} ({}_k p_x)^{\gamma_i}$$

$\gamma_i \geq 0$ is a survival pessimism index

Proposition 1

Suppose that various mild assumptions hold.

1. Survival beliefs: $\gamma_i = (>)(<) 1 \Leftrightarrow e_x^i = (<)(>) e_x$.
2. Perfect pessimism: as $\gamma_i \to \infty$, $e_x^i \to 0$.
3. Perfect optimism: $\gamma_i = 0 \Leftrightarrow e_x^i = \omega - x > 0$.
4. One-to-one correspondence: e_x^i is strictly decreasing wrt γ_i.

Subjective Mortality: Probability (p_x)-scaling

1-year survival probability scaling

$$p_x^i = \min(\nu_i\, p_x,\, 1)$$

Subjective life expectancy

$$e_x^i = \sum_{k=1}^{\omega-x} \prod_{j=0}^{k-1} \min(\nu_i\, p_{x+j},\, 1)$$

$\nu_i \geq 0$ is a survival optimism index

Proposition 2

Suppose that various mild assumptions hold.

1. Survival beliefs: $\nu_i = (>)(<)\, 1 \Leftrightarrow e_x^i = (>)(<)\, e_x$.
2. Perfect pessimism: $\nu_i = 0 \Leftrightarrow e_x^i = 0$.
3. Perfect optimism: $\nu_i \geq 1/\underline{p}_x \Leftrightarrow e_x^i = \omega - x > 0$ where $\underline{p}_x = \min\{p_{x+j} : j \in [0, \omega - x - 1]\}$.
4. Bounded one-to-one correspondence: e_x^i is strictly increasing wrt ν_i if $0 \leq \nu_i \leq 1/\underline{p}_x$ and e_x^i is constant at $\omega - x$ if $\nu_i \geq 1/\underline{p}_x$.

Life-cycle Model

- Classic life-cycle model following Campbell et al (2001), Cocco et al (2005). Our model also features annuities and social security
- Risk preferences: additive time-separable CRRA utility

$$\mathbb{E}\left[\sum_{k=1}^{\omega-x}\beta^{k-1}\left({}_kp_x^i\right)\frac{C_k^{1-\delta}}{1-\delta}\right]$$

${}_kp_x^i \Rightarrow$ boundedly rational survival expectations

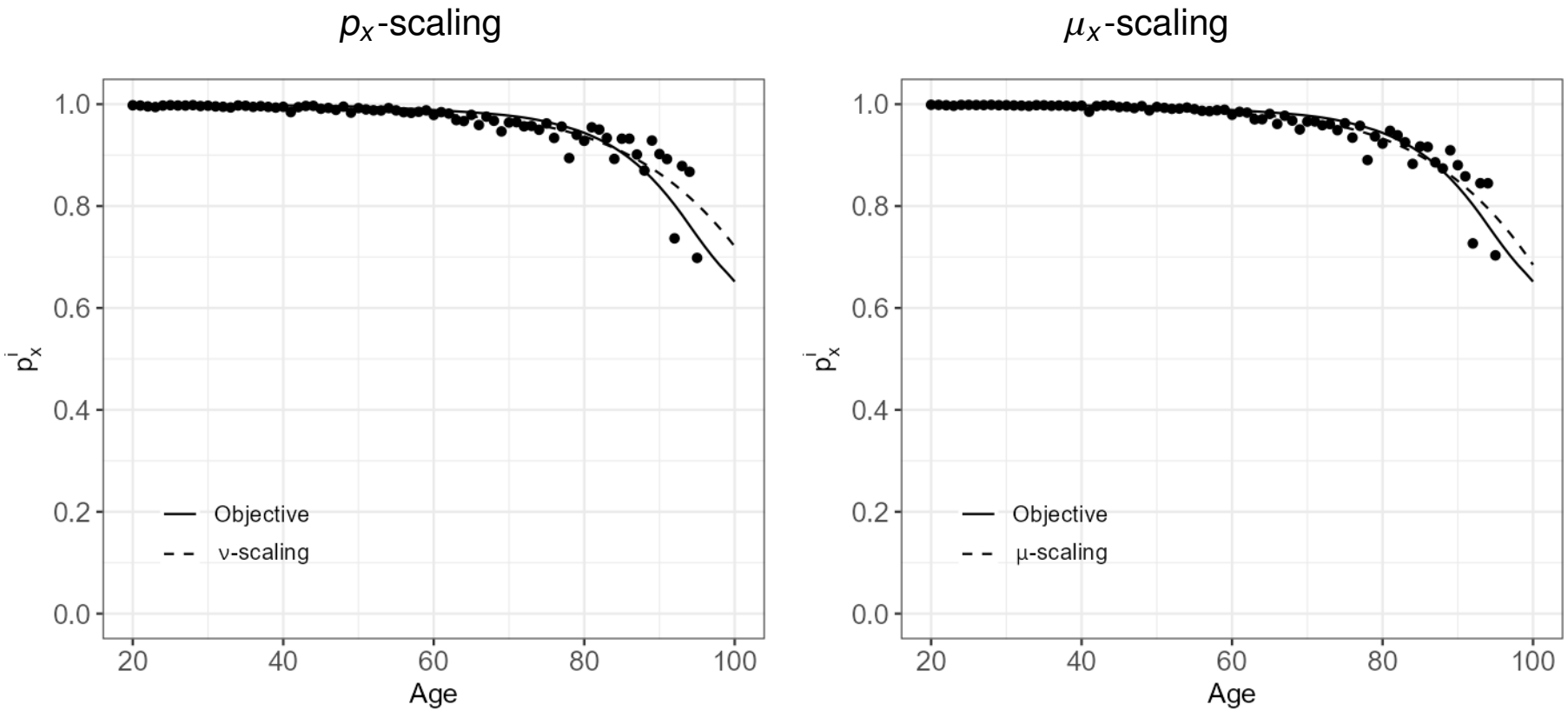
- Annuity priced using *objective* survival probabilities
- Portfolio decision: risk-free asset earning constant risk-free rate, risky asset earning Gaussian log-return
- Portfolio constraints: no short sales, no leverage
- Wages comprise: (1) deterministic function capturing hump shape income profile over age, (2) persistent productivity shock component, and (3) transitory shock
- Correlation between persistent productivity shock and risky asset return

Model Calibration: Life-Cycle Model

Parameterization: U.S. markets (Heimer et al, 2019; Love, 2013)
Baseline income profile: U.S. college graduates

Parameters	Value
Risk aversion	6
Discount factor	0.98
Risk-free rate	0.02
Equity Premium	0.04
Replacement rate	0.7567
Annuity Loading	0
Standard deviation of risky asset	0.18
Retirement age	65
Starting age	20
Terminal age	100
Uncertain income in retirement?	no
Bequest motive?	no

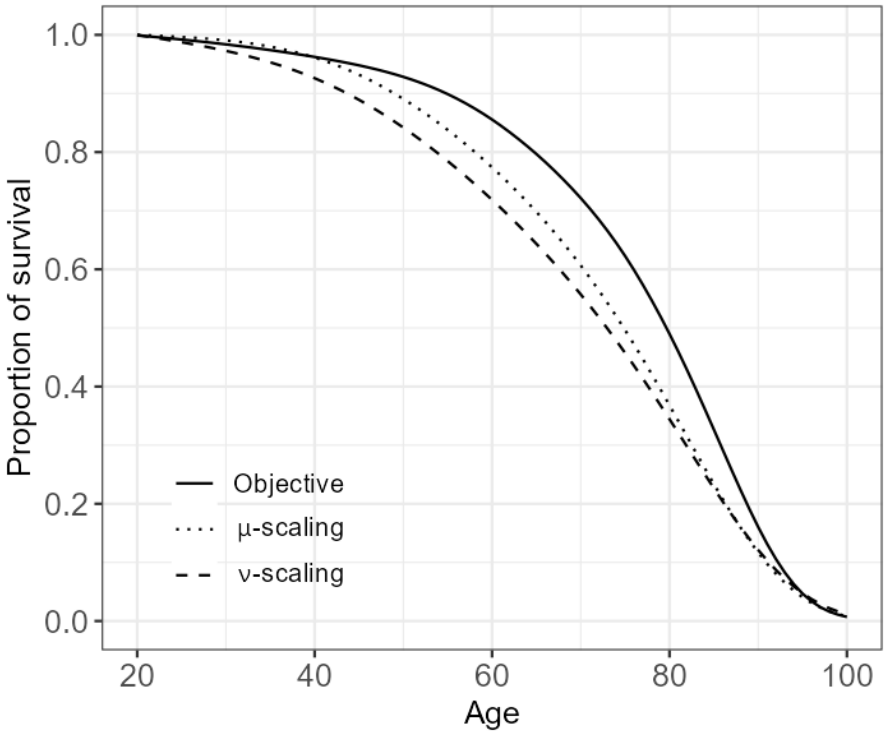
Model Calibration: Estimating Subjective Mortality



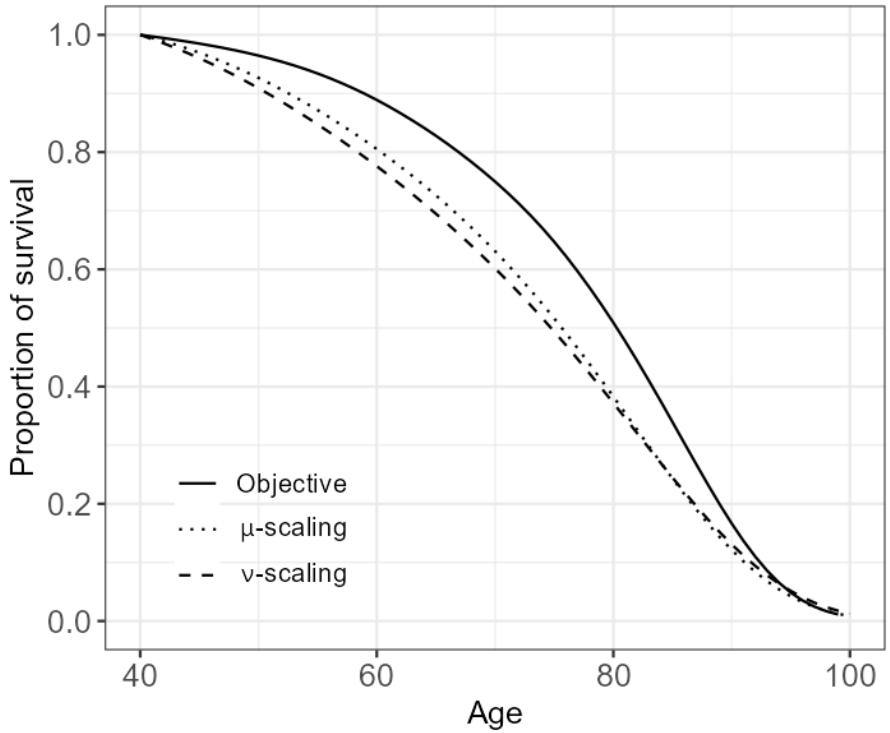
Solid line: objective 1-year survival probability from life table
Dots: 1-year survival prob. averaged over all individuals of a given integer age
Dashed line: spline curve fitted to dots, with monotonicity

Model Calibration: Survival Curves, I

At age 20

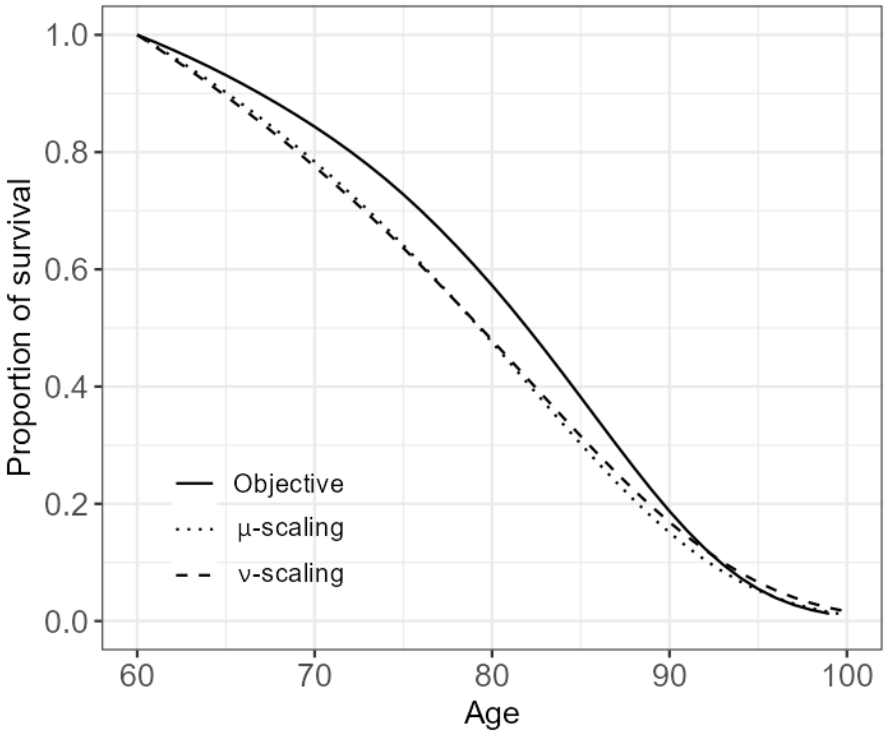


At age 40

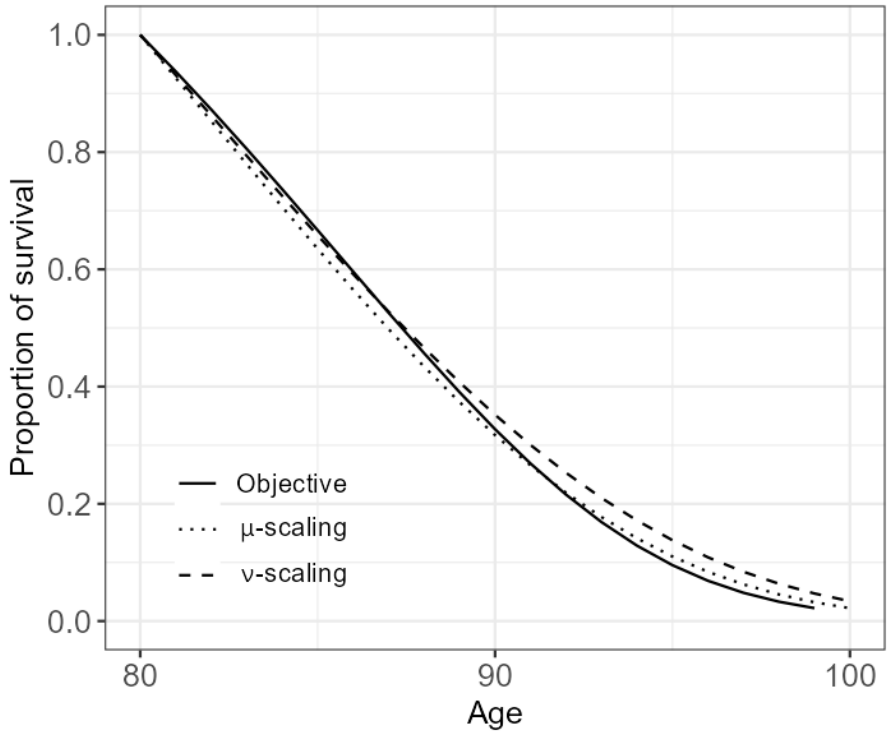


Model Calibration: Survival Curves, II

At age 60

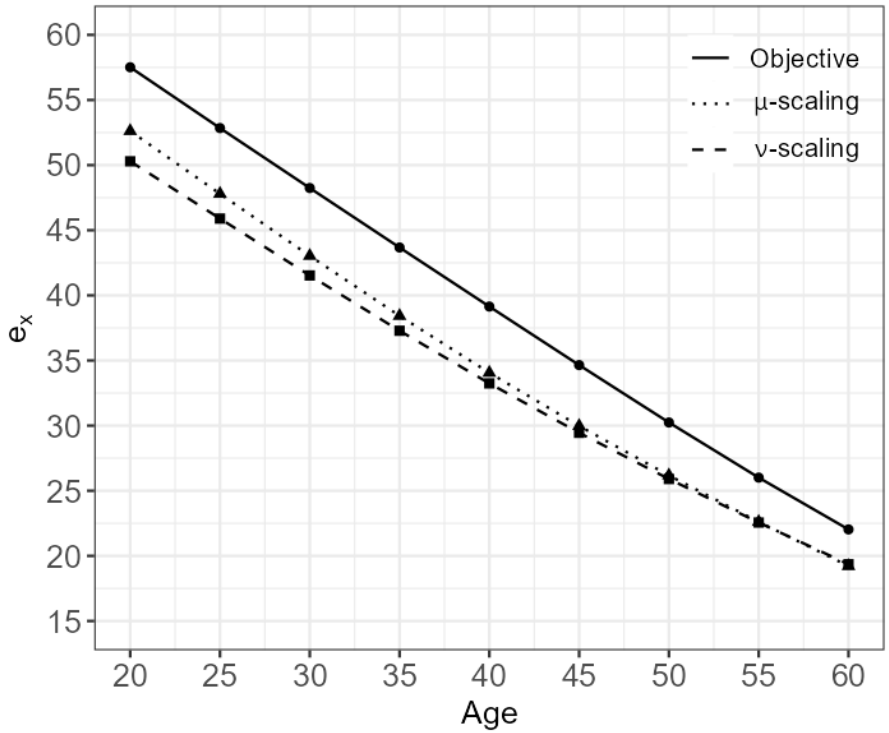


At age 80

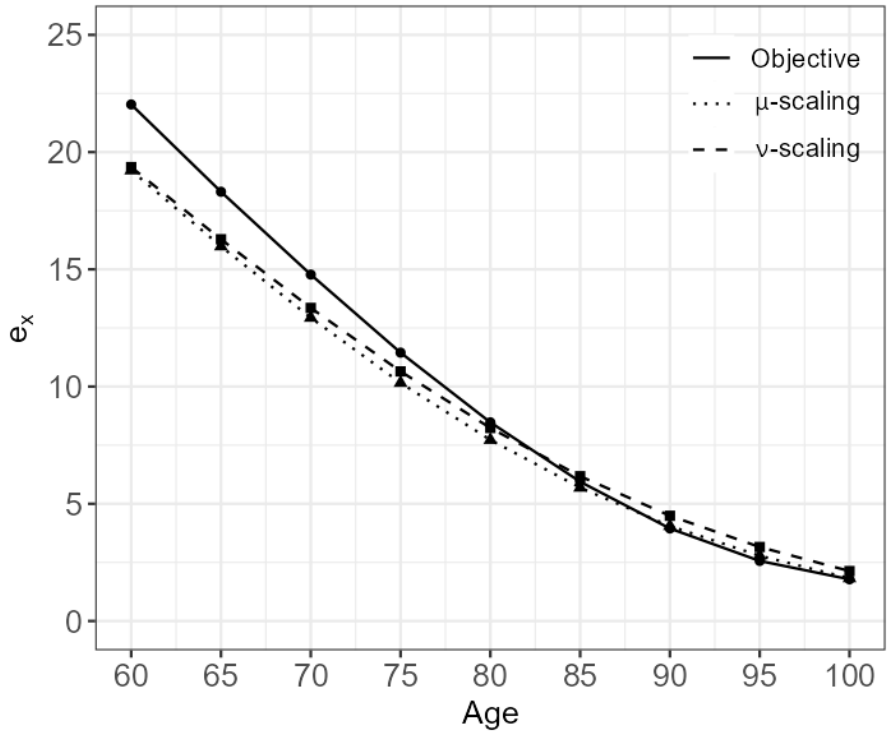


Model Calibration: Life Expectancy, I

At ages 20–60



At ages 60–100



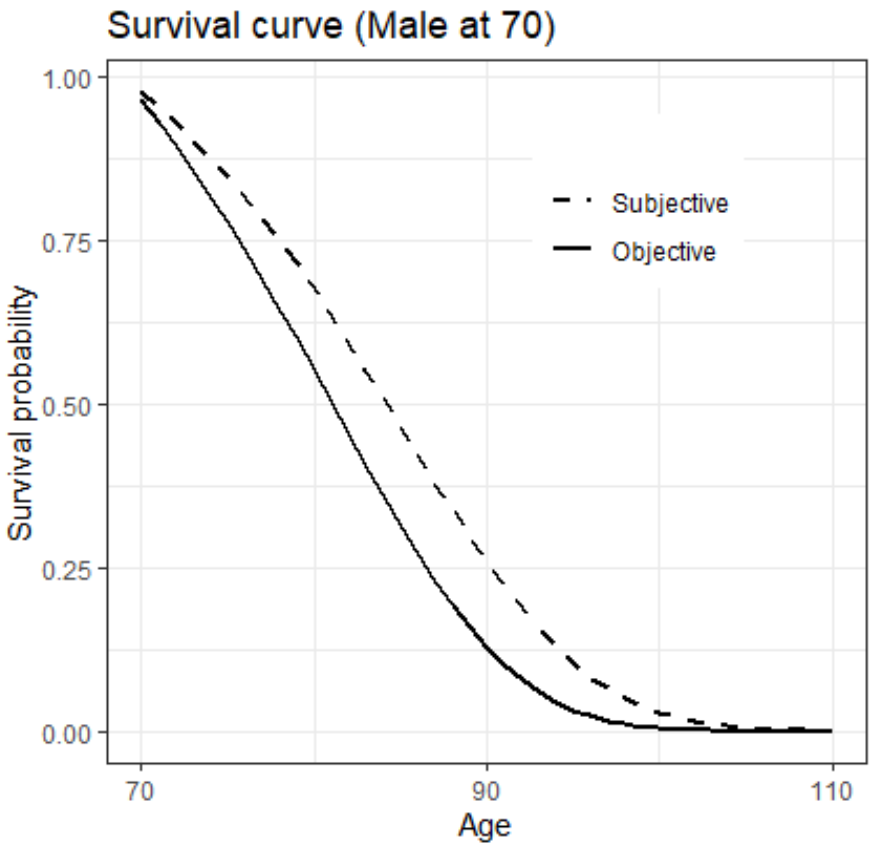
Model Calibration: Life Expectancy, II

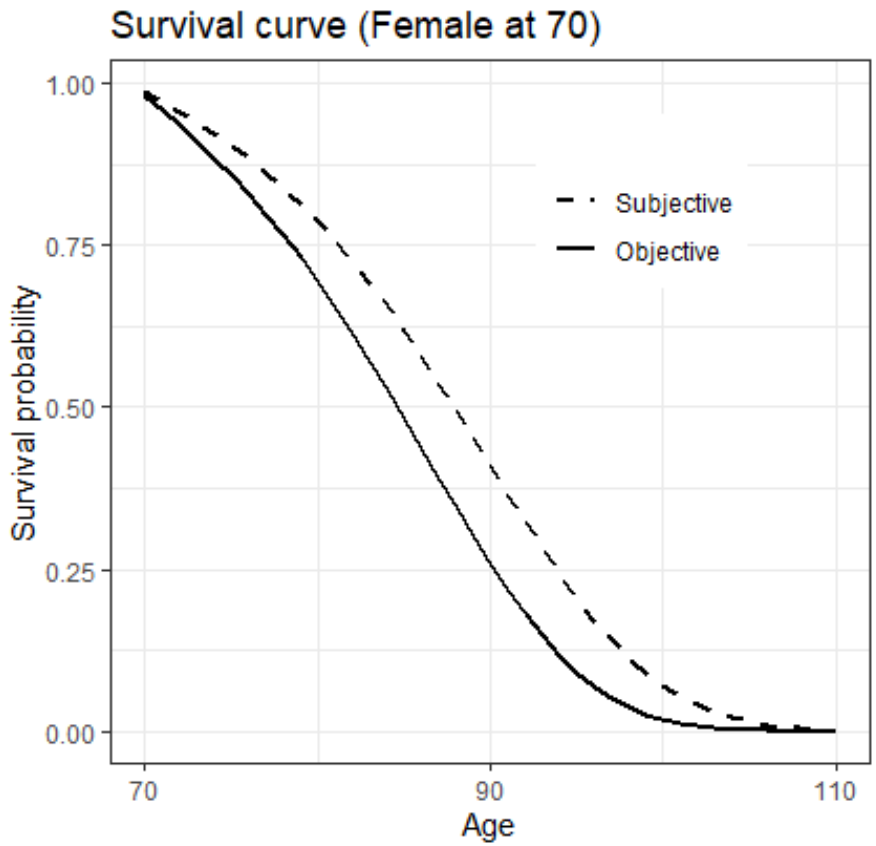
Life expectancy at different ages with objective and subjective survival probabilities (p_x-scaling and μ_x-scaling)

	e_{20}	e_{40}	e_{60}	e_{80}	e_{85}	e_{90}	e_{95}	e_{100}
Objective	57.51	39.14	22.03	8.47	5.94	3.95	2.56	1.78
p_x-scaling	50.29	33.24	19.36	8.23	6.18	4.45	3.16	2.13
μ_x-scaling	52.62	34.06	19.22	7.72	5.69	4.05	2.78	1.82

Model Calibration: Survival Curves, Gan et al (2015)

Survival curves by Gan et al (2015): old ages only

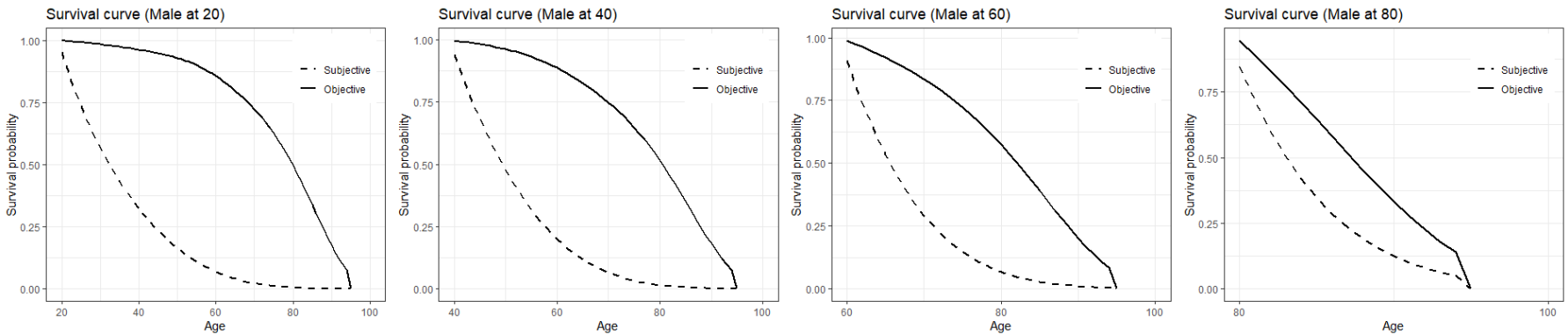




Model Calibration: Survival Curves, Heimer et al (2019)

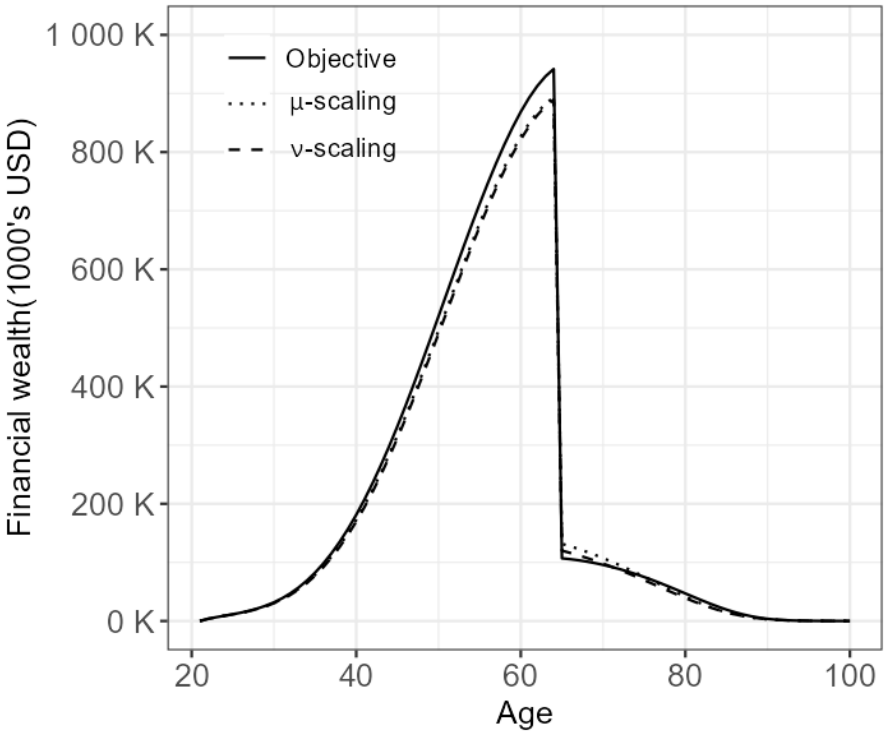
Survival curves by Heimer et al (2019) based on quadratic regression on age of 4 reported subjective survival probabilities

There is a large deviation between objective and subjective curves

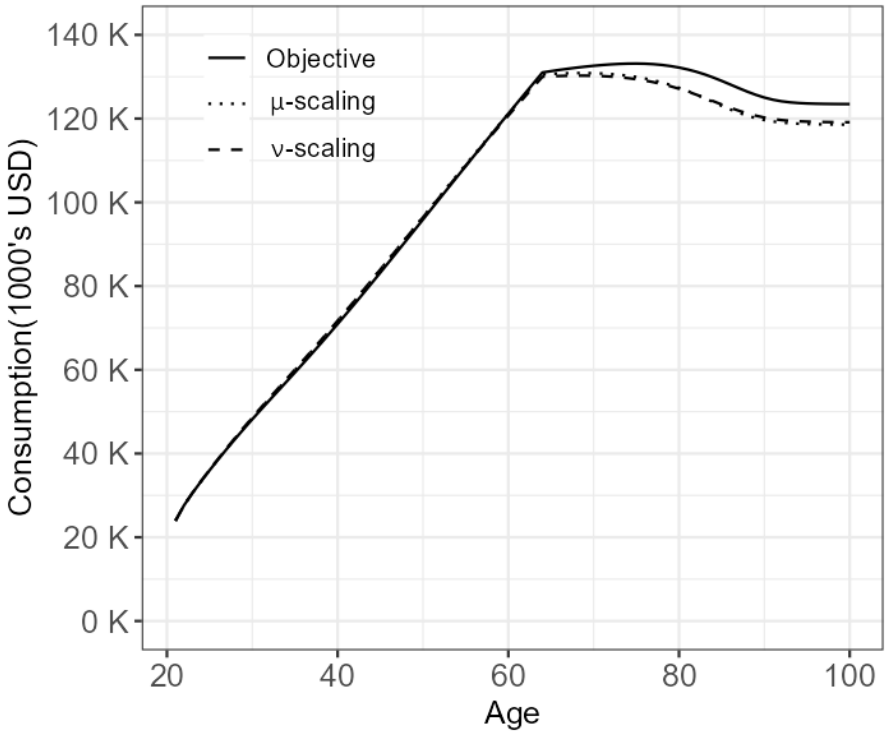


Results: Financial Wealth, Consumption (Average)

Financial wealth

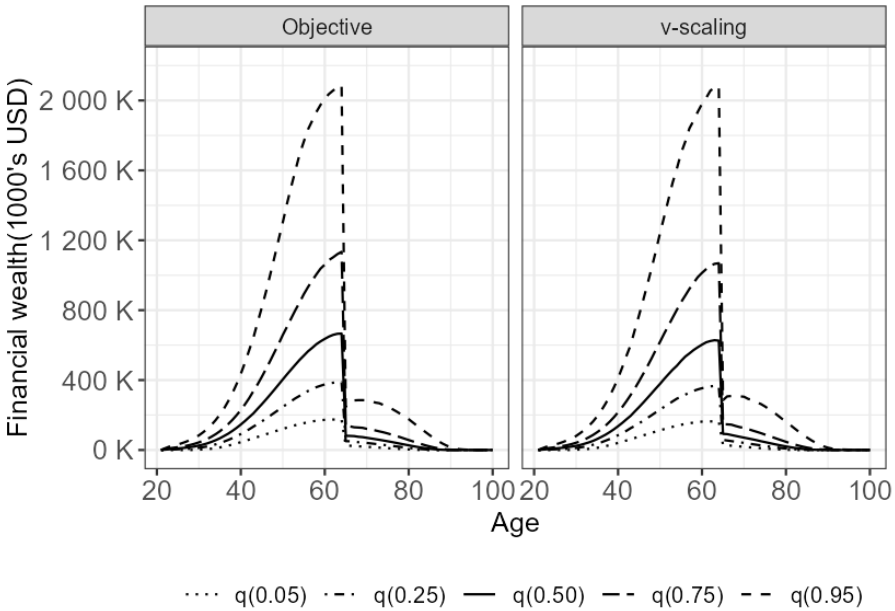


Consumption

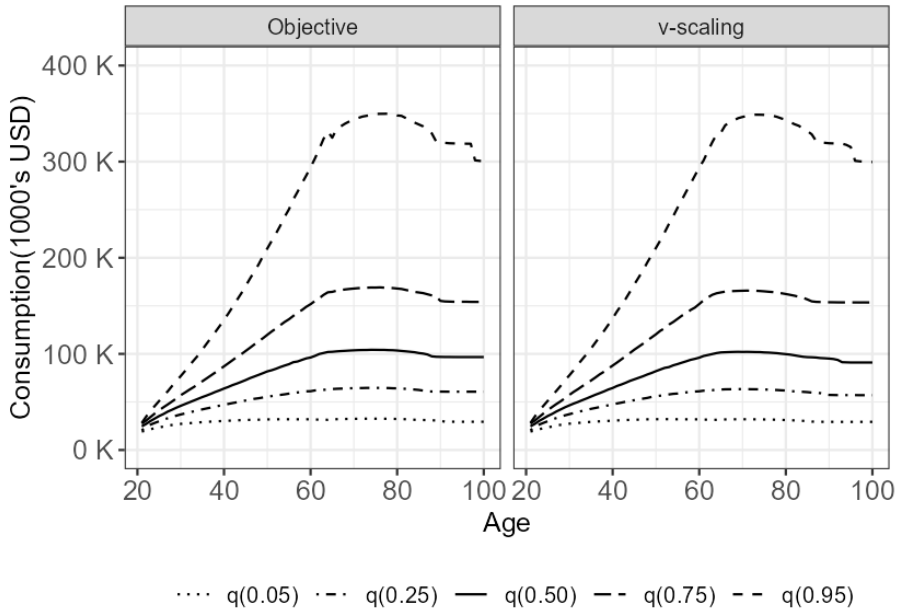


Results: Financial Wealth, Consumption (Quantiles)

Financial wealth

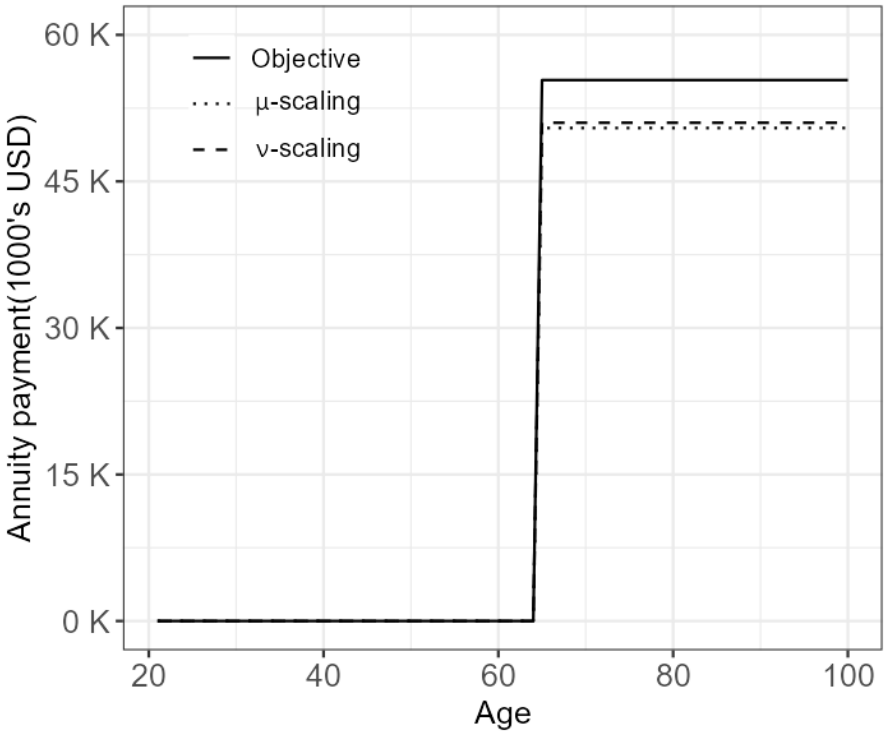


Consumption

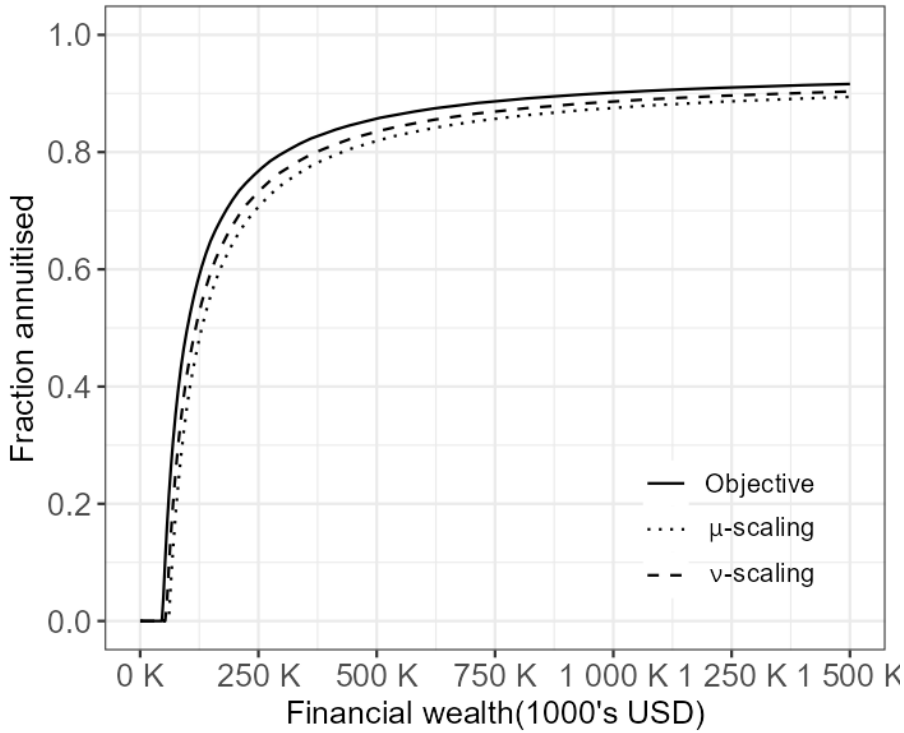


Results: Annuitization (Average)

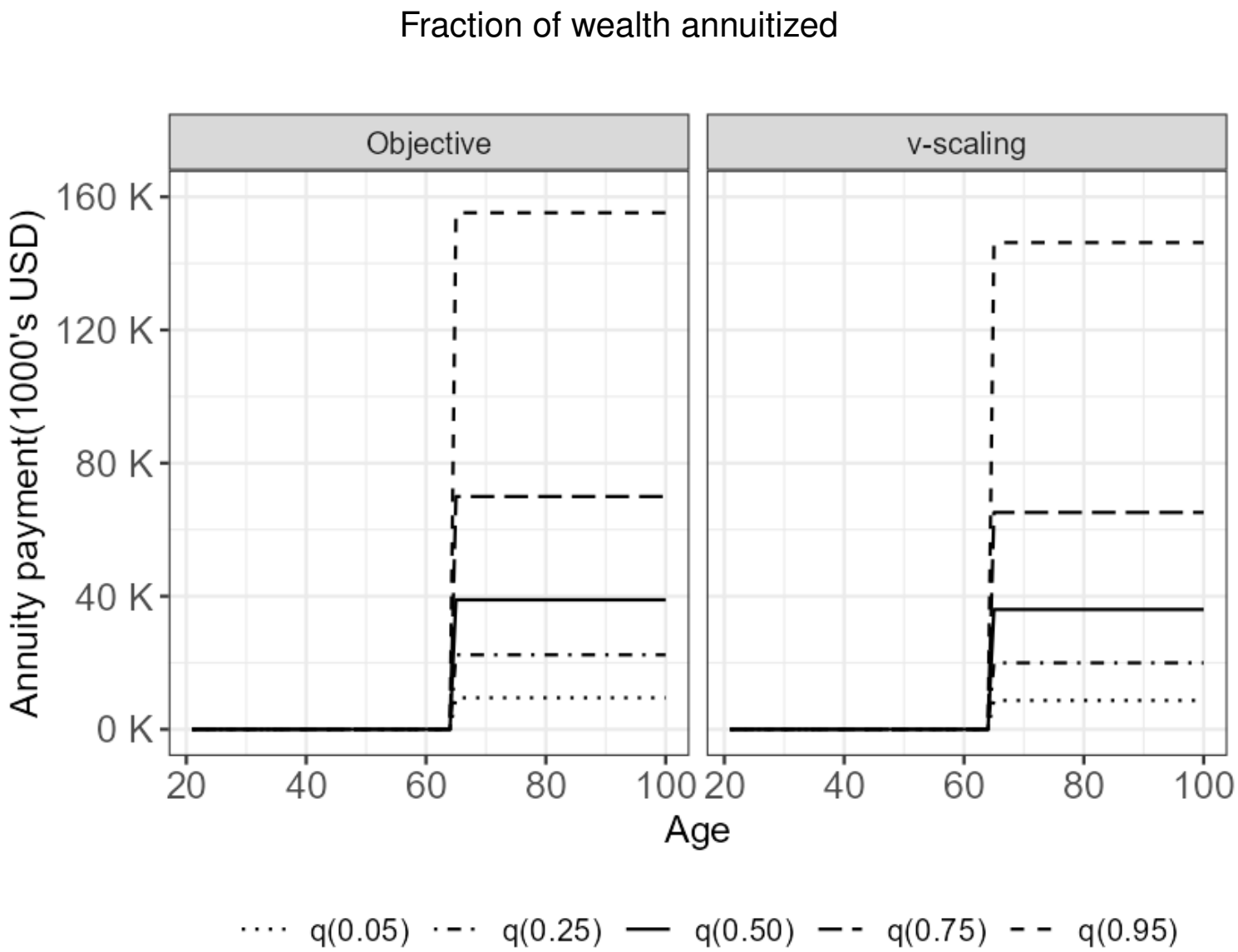
Annuity payment



Fraction of wealth annuitized



Results: Annuitization (Quantiles)



Results: Average

	Objective	p_x-scaling	μ_x-scaling
Consumption			
at 21	23,934	23,946	23,937
at 40	70,943	71,730	71,639
at 60	121,178	120,710	121,126
at 80	132,193	127,131	127,353
at 100	123,471	119,136	118,571
Financial wealth			
at 64	941,764	889,483	893,470
Annuity purchase			
at 65	837,959	771,858	763,702
Annuity payment			
after 65	55,373	51,005	50,466

Results: Robustness

These results are robust to variations in

- annuity loading
- labour income profile: college graduates, high school graduates, high school dropouts
- risk aversion coefficient
- time preference/discount factor

Conclusion

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Problem 1: Subjective mortality

How do we measure subjective mortality?

Answer:
We use stated subjective life expectancy e_x^i from a survey and scale objective mortality via

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- survival probability p_x

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Does subjective mortality explain:

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- under-annuitization?

Answer:
These stylized facts are only very marginally explained by subjective mortality

Selected References

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