A tour into the research on longevity risk

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Longevity risk

Risk of a longer lifetime than expected

For the individual: Risk of outliving his/her own wealth

For the provider: Lifelong payments, independent of

$$\underbrace{\text{Individual's lifetime}}_{\text{Random fluctuations}} \quad \& \quad \underbrace{\text{Average lifetime of the population}}_{\text{Aggregate fluctuations}}$$

Disclaimer

In the following

I will consider its risk management both in the individual's and providers' perspectives

However

Longevity risk management is a rather huge research area

I will outline the main research topics and I will cite (just) some contributions

In particular, touching the topics addressed by EP in his research

Longevity risk management for the individual – I

Alternatives about the post-retirement income

- Self-annuitization (Income drawdown)
- Reverse mortgage
- Group self-annuitization (GSA)
- Guaranteed Minimum Withdrawal Benefits (GMWB)
- Arrangements with a linking to longevity/mortality (tontines, mortality/longevity-linked annuities)
- Old age annuities (Longevity insurance), Extendable annuities, Ruin Contingent Life Annuity (RCLA)
- Guaranteed Minimum Income Benefits (GMIB)
- Fixed-amount and participating annuities

Different size of the longevity guarantee (and financial guarantees, as well), depending on the mortality credits

Longevity risk management for the individual – II

Choice of the post-retirement income: Annuity puzzle

Under appropriate assumptions, it would be optimal for an individual to annuitize (his/her whole wealth) at retirement

See Yaari (1965)

However: Individuals prefer not to annuitize or to delay annuitization

Several studies address the behaviour and preferences of individuals in this respect

For example: Lockwood (2012) Schreiber and Weber (2016) Peijnenburg et al. (2016) De Villiers-Strijdom and Krige (2023) Arandjelovic et al. (2023)

Longevity risk management for the individual – III

Optimal annuitization time and asset allocation

When not annuitizing, the mortality credit is lost, but this can be mitigated by higher investment returns

Delaying the annuitization decision increases the risk of ruin for the individual

See: Milevsky and Robinson (2000) Milevsky (2001) Milevsky (2005a) Milevsky and Young (2007a) Milevsky and Young (2007b) Gerrard et al. (2012) Bär et al. (2021) Liang and Young (2023)

Longevity risk management for the individual – IV

Post-retirement strategies and preferences

(Lifecycle) Asset portfolio choice, gradual annuitization, incentives to annuitize, combinations of benefits (annuities and death benefits, annuities and health-related benefits), optimal choices under subjective mortality beliefs, . . .

See: Brown (2001) Davidoff et al. (2005) Dus et al. (2005) Schmeiser and Post (2005) Milevsky and Young (2007a) Milevsky and Young (2007b) Horneff et al. (2008) Bayraktar and Young (2009) Horneff et al. (2010) Bruhn and Steffensen (2011) Hanewald et al. (2013) Maurer et al. (2013) MacDonald et al. (2013) Kling et al. (2014a) Gan et al. (2015) Maurer et al. (2016) Brown et al. (2017) Delong and Chen (2017) Chen et al. (2019a) Heimer et al. (2019) Apicella and De Giorgi (2023) Bär and Gatzert (2023) Chen et al. (2023) Chen et al. (2021a) O'Dea and Sturrock (2023)

Longevity risk management for the individual – V

A particular solution to fund the post-retirement income: Reverse mortgages

A loan on a residential property, to be repaid upon death

See: Shao et al. (2015) Fornero et al. (2016) Pearson and Lacombe (2021) Ho et al. (2022)

Longevity risk management for the individual – VI

Group Self-Annuitization and tontine solutions

- Self-insured pools
- Mortality credits, but not guaranteed
- Mitigation of random fluctuations, depending on the pool size and composition

See: Stamos (2008) Piggott et al. (2005) Qiao and Sherris (2013) Donnelly et al. (2013) Donnelly (2015) Milevsky and Salisbury (2016a) Bräutigam et al. (2017) Bernhardt and Donnelly (2019) Chen and Rach (2019) Chen et al. (2019b) Chen et al. (2020) Bernhardt and Donnelly (2021) Chen and Rach (2021) Chen et al. (2021b) Dagpunar (2021) Weinert and Gründl (2021) Hieber and Lucas (2022)

Longevity risk for the provider – I

Two risk components

Individual longevity risk: A process risk

- "Pooling" or insurance risk
- Reduced (relative) importance in face of: Homogeneity and large portfolio size, concentration of the pdf of the lifetime
- Subject to transfer through traditional reinsurance

Aggregate longevity risk (simply: longevity risk): A systematic risk

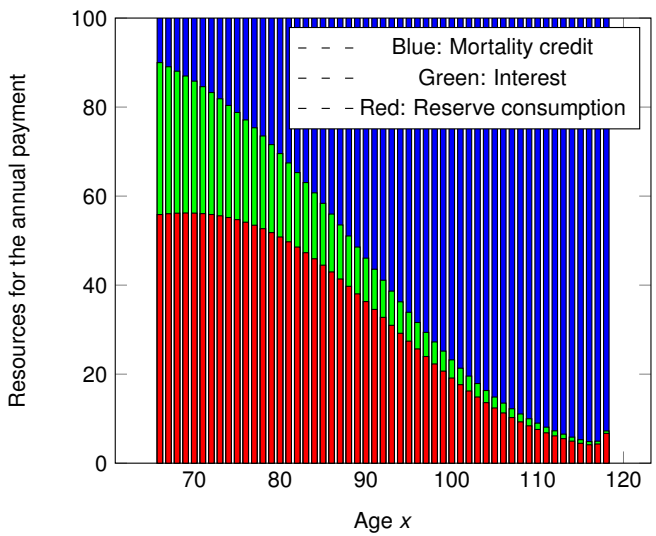
- Originated by the uncertainty of the mortality representation (model or parameter risk), in particular in respect of the modal age at death (and, then, the expected lifetime)
- Relative importance independent of the portfolio size $\Rightarrow$ Traditional risk transfer solutions are ineffective

Longevity risk for the provider – II

Timing of the longevity risk

From the recursion for the (individual) reserve of a fixed-benefit life annuity in arrears:

$$\underbrace{b}_{\text{Annual payment}} = \underbrace{V_t - V_{t+1}}_{\text{Reserve "consumption"}} + \underbrace{V_t \cdot i}_{\text{Interest}} + \underbrace{(V_{t+1} + b) \cdot q_{x+t}}_{\text{Mortality credit}}$$



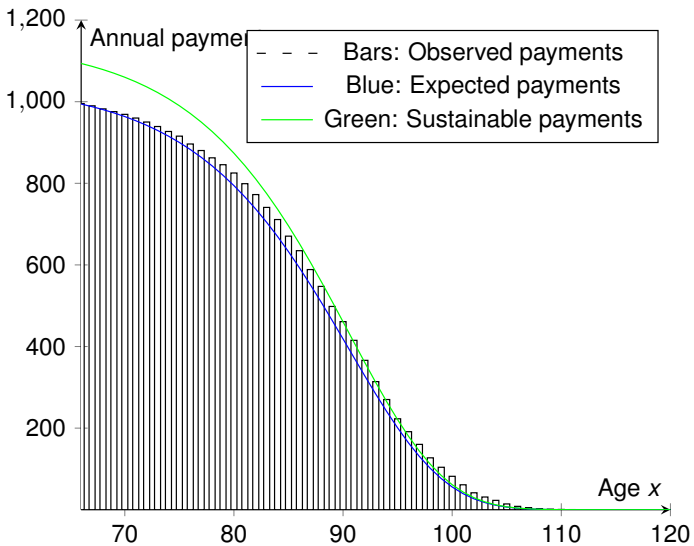
The impact of longevity risk becomes significant at later ages

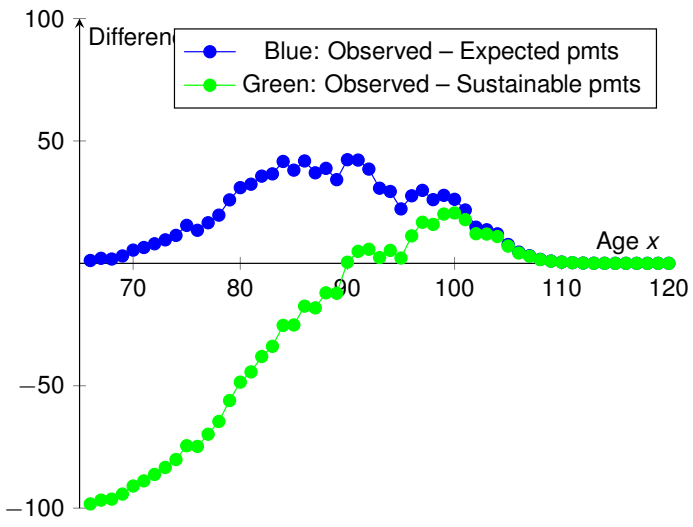
In particular:

- Aggregate longevity risk: Around the Lexis point
- Individual longevity risk: At the oldest ages

Risk management solutions – I

Example: Annual payments in a life annuity portfolio (one cohort)



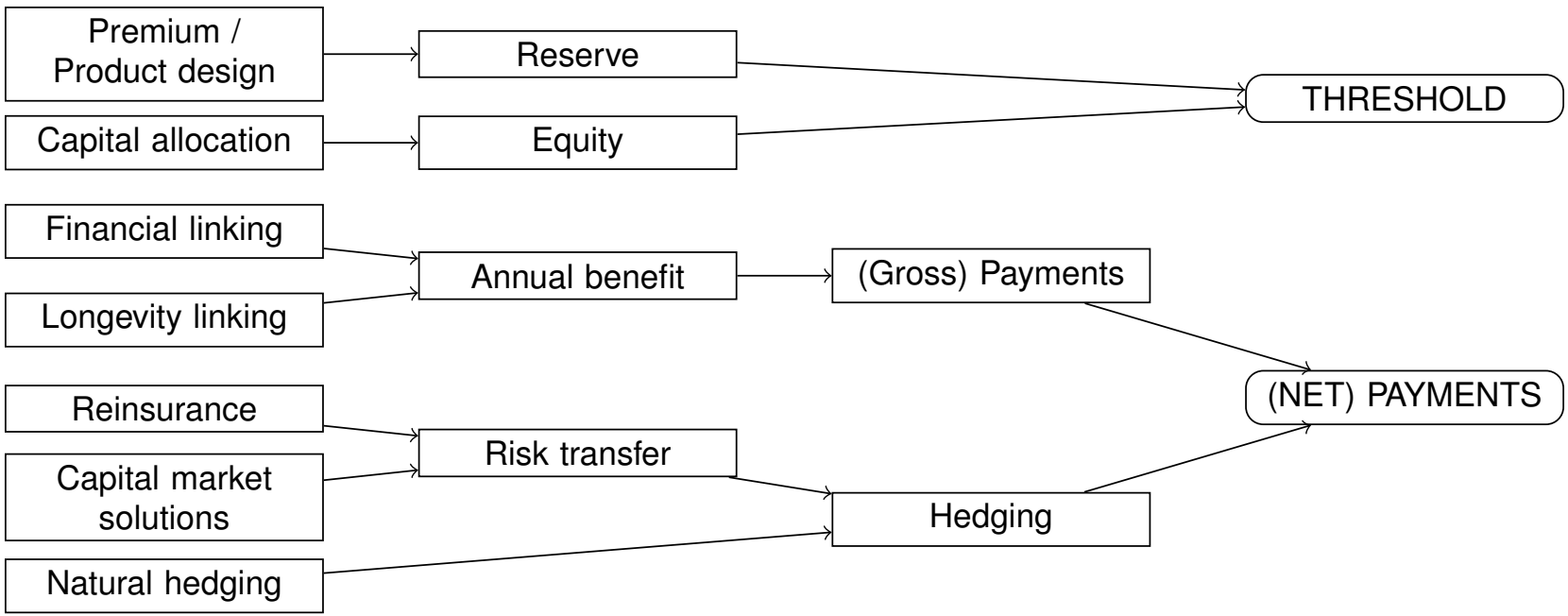


Possible targets

- Higher threshold, ore improved profile (i.e., amount of sustainable payments $\rightarrow$ green line)
- Reduced or smoothed profile of annual payments ($\rightarrow$ bars)

Risk management solutions – II

The possible actions



Pricing – I

Choice of the mortality model

Mortality modelling: A big chapter of actuarial research

Just to mention EP's main contributions on this topic:

- Survival models in a dynamic context: A survey (2004), Insurance: Mathematics & Economics (Pitacco (2004))
- Stochastic mortality: The impact on target capital (2009), ASTIN Bulletin (with AO) (Olivieri and Pitacco (2009))
- Modelling longevity dynamics for pensions and annuity business (2009), Oxford University Press (with M. Denuit, S. Haberman and AO) (Pitacco et al. (2009))

Pricing – II

Customized pricing, based on lifestyles or health conditions

Special rate (or underwritten) annuities

- Lifestyle annuity, Enhanced annuity, Impaired life annuity, Care annuity
- For a general description: Ridsdale (2012)

Consequence: Greater heterogeneity of the annuity portfolio

Implications of the heterogeneity of the population in respect of pricing: Meyricke and Sherris (2013)

For a possible risk classification approach, based on frailty models: Olivieri and Pitacco (2016)

Analysis of the portfolio risk profiles: Pitacco and Tabakova (2022)

Product design – I

Product design aimed at increasing the annuity rate

Time restrictions: Old age annuities (or Longevity insurance), Extendable annuities, Guaranteed Minimum Withdrawal Benefits (GMWB)

Old age and extendable annuities: Milevsky (2005b) Huang et al. (2009) Gong and Webb (2010) Rocha et al. (2011) Nadine Gatzert and Schmeiser (2012) Gatzert and Klotzki (2016) Donnelly and Young (2017) Horneff et al. (2020) Olivieri and Pitacco (2022)

GMWB: Next slide

The evolving structures of the longevity guarantee in life annuities

For a general description: Pitacco (2016)

Product design – II

Variable annuities

Wide range of life insurance products, offering a number of guarantees

Several contributions, focussing on the pricing and valuation of the guarantees (many, on financial guarantees)

Main features: Ledlie et al. (2008)

General valuation frameworks: Bauer et al. (2008) Bacinello et al. (2011)

Considering stochastic mortality: Fung et al. (2014) Gudkov et al. (2019) Bacinello et al. (2023), and references therein

A special case (significant in the past): The Guaranteed Annuity Option (GAO). Considering stochastic mortality: Ballotta and Haberman (2006) Biffis and Millossovich (2006) Kling et al. (2014b)

Product design – III

Longevity linking

The annual amount is updated depending on a longevity index $\Rightarrow$ Participation to longevity profits/losses

Structure similar to tontine and self-insured solutions, but providing guarantees (i.e., mortality credits partially guaranteed)

Longevity-linked arrangements: Richter and Weber (2011) Denuit et al. (2011) Denuit et al. (2015) Bravo and de Freitas (2018) Olivieri and Pitacco (2020a,b)

Modern structure of tontine annuities: Weinert and Gruendl (2008) Milevsky (2014) Milevsky and Salisbury (2015) Milevsky and Salisbury (2016b) Chen et al. (2019b) Chen and Rach (2019) Chen et al. (2020) Chen and Rach (2021)

Equity

Capital required either based on:

- Standard formulae or (partial) internal models
- Own assessment
- Deterministic vs stochastic models

A significant issue: Cost of capital and shareholders' value

Further: Cost of capital vs cost of risk transfer solutions

Risk transfer solutions vs required capital: Barrieu and Loubergé (2013) Meyricke and Sherris (2014) Boon et al. (2020)

Also considering shareholders' value: Gruendl et al. (2006) Blackburn et al. (2017)

Solvency investigations: Hària et al. (2008) Olivieri and Pitacco (2009) Olivieri (2011) Coppola and D'Amato (2012)

Also considering shareholders' value: Nirmalendran et al. (2014)

Natural hedging – I

Across Lines of Business (LOBs)

Mitigation effects when matching life annuity liabilities with life assurance liabilities (however: different range of ages ($\Rightarrow$ differences in the mortality dynamics) & different magnitude of the value of liabilities)

See: Cox and Lin (2007) Ludkovski and Bayraktar (2009) Cox et al. (2013) Gatzert and Wesker (2014) Wong et al. (2017)

Across time

Packaging life annuity with death benefits in one product

Not very effective as a solution to longevity risk, as the death benefit is usually limited to not very high ages

In practice, a matter of bequest preferences

Natural hedging – II

Across populations

Compensate differences in the mortality dynamics of different populations (in particular, in different geographical areas)

See: Li et al. (2015) Luciano et al. (2015) Biffis et al. (2017) Boonen et al. (2017)

Risk transfer – I

Capital market solutions

Longevity swaps, longevity bonds: Derivatives contingent on a longevity index (Insurance-Linked Securities – ILS – or longevity-linked securities)

Very rich research area (in particular, since when the aggregate longevity risk became significant). However, the market remains underdeveloped (basis risk and long-term exposure to risk)

A comprehensive reference: Blake et al. (2023), and previous editions

Risk transfer – II

Reinsurance

Traditional arrangements are not effective, due to the non-pooling nature of the (aggregate) longevity risk

Specific arrangements for longevity risk typically resemble the structure of longevity swaps, with a lower magnitude of the basis risk

Reinsurance is indemnity-based (whereas longevity swaps and bonds are index-based) $\Rightarrow$ The basis risk is reduced (when not negligible), and the solution can be more expensive than ILS

Index-based vs Indemnity-based hedging strategies: References in Blake et al. (2023)

EP's research on longevity risk and annuities – I

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- with A.O.: Rischio demografico e riassicurazione per portafogli di rendite vitalizie, Working Paper No. 12, CERAP - Università Bocconi, Milano

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- Tavole di mortalità proiettate e loro impiego in ambito attuariale. Appunti introduttivi, Quad. No. 1/1997 del Dipartimento di Matematica Applicata alle Scienze Economiche, Statistiche ed Attuariali, Università di Trieste

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- **with P. Marocco: Longevity risk and life annuity reinsurance, Transactions of the 26th International Congress of Actuaries, Birmingham, vol. 6: 453–479**
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2001

- Modelli previdenziali nel nuovo scenario demografico. In: Editoriale Generali, Fondi pensione e sviluppo economico, Convegno Industria e Assicurazione, Villa Manin di Passariano (UD): 159-177
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- with A.O.: Rendite vitalizie: longevity risk, garanzie demografiche, profili tariffari, Working Paper No. 22, CERAP - Università Bocconi, Milano

2002

- with A.O.: Inference about mortality improvements in life annuity portfolios, Transactions of the 27th International Congress of Actuaries, Cancun, Mexico
- Un'introduzione al longevity risk. In: Liber Amicorum per Alessandro Di Lorenzo, Dipartimento di Matematica e Statistica, Università Federico II, Napoli: 309-323
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- with A.O.: Life annuities and longevity dynamics, Working Paper No. 36, CERAP - Università Bocconi, Milano

2007

- Mortality and longevity: a risk management perspective, Invited lecture at the 1st IAA Life Section Colloquium, Stockholm

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- **with A.O: Assessing the cost of capital for longevity risk, Insurance: Mathematics & Economics, 42 (3): 1013-1021**

EP's research on longevity risk and annuities – II

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2014

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- High Age Mortality and Frailty. Some Remarks and Hints for Actuarial Modeling, CEPAR Working Paper 2016/19, University of New South Wales, Sydney
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- **with A.O: Linking annuity benefits to the longevity experience: Alternative solutions, Annals of Actuarial Science, 14(2), 316-337**
- with A.O.: Longevity-Linked Annuities: How to Preserve Value Creation Against Longevity Risk. In: Borda M., Grima S., Kwiecien I. (eds) Life Insurance in Europe. Financial and Monetary Policy Studies, vol 50. Springer, Cham

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- **Life annuities. Risk Books**

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- **with D. Tabakova: Special-rate life annuities: Analysis of portfolio risk profiles, Risks, 10(3), 65**
- **with A.O: Time Restrictions on Life Annuity Benefits: Portfolio Risk Profiles, Risks, 10(8), 164**

Thank you! 🙏

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